6.2 Double Angle Identities

① $\sin(2A) = \sin(A+A)$

$= \sin A\cos A + \cos A\sin A$

$= \sin A\cos A + \sin A\cos A$

$$\boxed{\sin(2A) = 2\sin A\cos A}$$

② $\cos(2A) = \cos(A+A)$

$= \cos A\cos A - \sin A\sin A$

$$\boxed{\cos(2A) = \cos^2 A - \sin^2 A}$$

$\cos^2 A + \sin^2 A = 1$

$\sin^2 A = 1 - \cos^2 A$

$= \cos^2 A - \boxed{1 - \cos^2 A}$

$$\boxed{\cos(2A) = 2\cos^2 A - 1}$$

$\cos^2 A = 1 - \sin^2 A$

$= \boxed{1 - \sin^2 A} - \sin^2 A$

$$\boxed{\cos(2A) = 1 - 2\sin^2 A}$$

③ $\tan(2A) = \tan(A+A)$

$= \dfrac{\tan A + \tan A}{1 - \tan A \cdot \tan A}$

$$\boxed{\tan(2A) = \frac{2\tan A}{1 - \tan^2 A}}$$

Ex1) Write $\cos^2(\pi/3) - \sin^2(\pi/3)$ as a S.T.F.

Recall $\cos^2 A - \sin^2 A = \cos 2A$

$\cos(2\pi/3)$

Ex) Consider $\frac{1-\cos(2x)}{\sin(2x)}$.

a) Write the NPV's.

$\sin(2x) \neq 0$

$\frac{2x}{2} \neq \frac{\pi n}{2}, n \in Z$

$x \neq \frac{\pi}{2} n, n \in Z$

b) Write $\frac{1-\cos(2x)}{\sin(2x)}$ as a S.T.F.

$\cos 2A = 1 - 2\sin^2 A$

$\frac{1-(1-2\sin^2 x)}{2\sin x \cdot \cos x}$

$\frac{2\sin^2 x}{2\sin x \cdot \cos x}$

$\frac{\cancel{2}\,\cancel{\sin x} \cdot \sin x}{\cancel{2}\,\cancel{\sin x} \cdot \cos x}$

$\tan x \rightarrow$ NPV's $x = \frac{\pi}{2} + \pi n, n \in Z$

c) Graph both expressions and compare.

Ex) Write $12\sin\theta\cos\theta$ as a S.T.F.

$6(2\sin\theta\cos\theta)$

$\hookrightarrow 6\sin(2\theta)$

$\boxed{\text{Pg. 306-308 \#1ce, 2c, 3-5, 11, 12, 14-16, 18, 20cd, C2}}$